Disc 4/16

Integration Pt. 1: Area

Goal: look at areas under curves.

$f(x) = x^2 \quad [0,4]$

1) Right Endpoint, $\Delta x = 1$

Area $\approx 1(1) + 1(4) + 1(9) + 1(16)$

2.) left endpoint:

Area $\approx 1(0) + 1(1) + 1(4) + 1(9)$

3.) midpoint.

$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$ ⤳ "tighter rectangles get closer to curve"

Problems

1.) Find the antiderivatives of

i) $4\sqrt{x} - 6x^2 + 3$

ii.) $\frac{1}{x} + \frac{1}{x^2+1}$

2.) Find the area under $\ln(x)$ on $[1,5]$ using left ~~and right~~ endpoint approx w. $\Delta x = 2$, $\Delta x = 1$, $\Delta x = 1/2$.

3.) 4.) Find the left endpoint approx. on $[0, 5]$ for $f(x) = x$, $\Delta x = 2$, $\Delta x = 1$, $\Delta x = 1/2$.

B.) Notice that we may write the area as. $\sum_{i=1}^{n} (\Delta x_i) x_i$. If $\Delta x_i = 5/n$ for all $i = 1, 2, \ldots n$, what is x_i?

$x_i = i(5/n)$.

C.) write this x_i into the formula $\left(\sum_{i=1}^{n} (5/n)(5/n) i = 25/n^2 \sum_{i=1}^{n} i\right)$.

Use that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ to show that

$\lim_{n \to \infty} \sum_{i=1}^{n} (\Delta x_i) x_i = 25/2$.

D.) Does this match the area under x as a triangle?